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THE PROPERTY OF THE PROBLEM OF REACTION DIFFUSION WITH DOUBLE NONLINEARITY AT THE GIVEN INITIAL CONDITIONS

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ABSTRACT

In work are received the oretical results on blow-up for a class of problems with additional linear and nonlinear diffusion and it would be interesting to understand reaction-diffusion profiles for these problems as well. Method of characteristics does not directly apply to such problems due to the diffusive nature of the dynamics. Special research techniques of the nonlinear parabolic equations which allow conducting rather detailed research blow up of solutions of a heat conduction equation with a source were developed.

***KEYWORDS:** Nonlinearity, Diffusion & Reaction*

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Original Article

1. INTRODUCTION

In recent years began to pay special attention to the unlimited decisions which are the reason of existence of an energy release, chemical reaction, etc. Such decisions arise in many physical processes (for example, combustion). In this regard in recent years the theory of blow up of decisions strongly develops, many works of A.A. Samarsky, S.P. Kurdyumov, A.P. Mikhaylov, V.A. Galaktionov, S.N. Dimova and many foreign scientists are devoted to this question. Blow up of the decision were called by decisions with the aggravation mode. Special research techniques of the nonlinear parabolic equations which allow to conduct rather detailed research blow up of solutions of a heat conduction equation with a source [1, 2, 3] were developed.

The group of the Bulgarian scientists, in particular S.N. Dimova, M.S. Kaschiyev, M.G. Koleva and D.P. Vasileva enter received important scientific result recently. In this model at a HS mode approximation with aggravation to the S-mode the possibility of existence of waves with the complex structure of the organization which are also the structures attractors described by invariant and group decisions is open. Half-width of these structures waves grows over time. It was supposed earlier that the difficult world of structures corresponds only to the LS mode with the reduced half-width at the prevailing role of action of nonlinear sources in comparison with diffusion processes. And in S.N. Dimova's work with colleagues mentioned above still the difficult world of the solitonic structures waves keeping the form at the growing half-width [3] is open.

In article [4] are proved an existence theorem and uniqueness of the generalized solution of an initial value problem for the equation

$$u_t = [\varphi(u_x)]_x + \psi(u).$$

Boundary value problems for the similar equations are investigated, for example, in the book [5] where it is possible to find further references.

Presence at an evolutionary task of the unlimited decision means its global insolubility on time (a non-existence of a global solution). In this case at the expense of an intensive energy release process of combustion can happen in the so-called mode to aggravation. In other words, the initial value problem has no global solution on time and in some instant $t = T_0 < +\infty$ aggravation moment), amplitude of the solution becomes infinitely big:

$$\sup_{x\in R^N} u(t,x) \to +\infty,\ \mathrm{t} \to \mathrm{T}_0^- .$$

At the same time up to the aggravation moment at all $0 < t < T_0$ non-negative continuous solution of a task is limited and also generalized. It cannot have all derivants, however function $|\nabla u(t,x)|$ is continuous everywhere in the area $(0,T_0)\times R^N$. From the physical point of view it means the continuity of a heat flux equal $W = -|x|^m |\nabla u|^{n-1} \nabla u$.

The nonlinear effect of final rate of propagation of temperature indignations was for the first time found by Zeldovich Ya.B. and Kompaneets A. S. [6], and effect of spatial localization of shift indignations – Martinson L.K. and Pavlov K.B. [7]. On examples of exact self-similar decisions in [8] it was shown that shift indignations can extend with a final speed unlike the incompressible Newtonian (*n=1*) and pseudoplastic fluids (*n<1*), in which the speed of distribution of shift indignations is infinite.

Kalashnikov A.S. [9] established the phenomenon of full cooling for final time in case of strong absorption $(0<\beta<1)$.

In [10] studied the following Cauchy problem:

$$\frac{\partial}{\partial t}\left(\rho(|x|)u\right) = \sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left(u^{m-1}|Du|^{p-2} u_{x_i}\right) \qquad (1)$$

in $Q_T = \mathbb{R}^N \times (0,T), N \ge 1,$

$$u(x,0) = u_0(x),\ \ x\in \mathbb{R}^N,\ \ u_0(x) \ge 0\ \ \textit{for constant}\ x\in \mathbb{R}^N \ . \qquad (2)$$

In the case of fast diffusion $0 < m + p - 1 \le 2$ estabilished bilateral evaluations L_∞.

Here

$$u = u(t,x),\ x = (x_1,\dots,x_N),\ |x| = \left(x_1^2 + \dots + x_N^2\right)^{\frac{1}{2}},\ Du = \left(u_{x_i},\dots,u_{x_i}\right) \qquad (3)$$

$$m + p - 3 > 0, \quad p - 1 > 0$$

$\rho(s), s \ge 0$ – decreasing, continuous, positive function; $\rho(0) = 1$

Besides,

$$supp u_0 \in B_{R_0} \equiv \{|x| < R_0\}, \ \ u_{0_{\infty \mathbb{R}^N}} < \infty \tag{4}$$

Typical example of the function ρ is

$$\rho = (1+|x|)^{-l}, \quad l > 0. \tag{5}$$

It is said that a solution of equation (1) has a finite speed of propagation property (FSP) if the perturbation of the conditions supp$u(., t_0) < \infty$ at some point in time $t_0 \geq 0$ follows that this property remains for all instants $t > t_0$ Otherwise say that the carrier of the solution (1) collapses for a short time (ShTC). Main objective of this work is clarification of conditions on $\rho(|x|)$, at which ShTC properties for the solution of a task (1), (2) take place. Let's note that if $\rho \equiv 1$, then, by virtue of conditions (3) and (4), solutions of problem (1), (2) have the property finite velocity of propagation. However if, for example, in (5) l "it is too big", then takes place ShTC.

In [11] the quasilinear degenerating parabolic equation with the non-uniform density investigated. It established that depending on behavior of density on infinity for the solution of an initial value problem take place or property of final rate of propagation of indignations, or destruction of the carrier for final time. The following second mixed problem is considered:

$$u_t - div\left(u^{m-1}|Du|^{p-2} Du\right) = 0 \quad in \quad Q_T = \Omega \times (0,T), \tag{6}$$

$$u^{m-1}|Du|^{p-2}\frac{\partial u}{\partial \vec{n}} = 0 \quad on \quad \partial\Omega \times (0,T) \tag{7}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{8}$$

where $\Omega \in \mathbb{R}^N$, $N \geq 2$ - unlimited domain, $mes_N \Omega = |\Omega|_N = \infty$; $\partial\Omega -$ noncompact rather smooth border Ω; $\vec{n} -$ external single normal to $\partial\Omega \times (0,T,)$, $T > 0$. Are supposed to $m + p - 3\langle 0,\ p-1\rangle 0,\ m+p-2 > max\left\{0, 1-\frac{p}{N}\right\}$; $u_0(x) \geq 0\ x \in \Omega\ and\ u_0 \in L_{1,loc}(\Omega)$ It is known [12] that at $m + p - 3 < 0$ (6) fall into to the equations describing process with fast diffusion..

In article [13] the initial value problem of rather parabolic equations with double nonlinearity of the following look is considered: $u_t = div\left(u^\alpha |Du|^{m-1} Du\right) + u^p$, where $0 < m + \alpha \leq 1$. It establishes the existence and non-existence on the whole of the time of solving this problem for initial data, slowly approaching zero.

The following Cauchy problem is considered:

$$u_t = div\left(u^\alpha |Du|^{m-1} Du\right) + u^p \tag{9}$$

$$(x,t) \in Q_T = \mathbb{R}^N \times (0,T), \ T > 0, \ N \geq 1$$

$$u(0,x) = u_0(x), \quad x \in \mathbb{R}^N \tag{10}$$

Here it assumed that $m+\alpha\le 1, m>0, m+\alpha>\max\left\{0,1-\frac{m+1}{N}\right\}, p>1$ and $u_0(x)$ – it is non-negative measurable function from a class $L_{1,loc}\left(\mathbb{R}^N\right)$. At $m+\alpha-1<0$ equality (9) without source falls into to the equations of fast diffusion, and at $m = 1$ it arises in a plasma physics [12]. The characteristic feature of this class of the equations is the lack of property of final rate of propagation of indignations. It is well known [14] that even in the simplest cases $m=1,\ \alpha=0$ there are solutions (9) which beyond all bounds grow for of course time, that is exists $T>0$, such that

$$u(\cdot,t)_{\infty,\mathbb{R}^N}\to\infty,\ \ t\to T<\infty.$$

It was shown that if $p<p^*=1+2/N$, that any non-negative solution of a task (9), (10) "blows up" for final time. If $p>p^*$ for initial functions, enough small in a sense, there is a decision in general on time. If $p=p^*$ joins in a mode case with aggravation.

2. PROBLEM DEFINITION

In the present work questions of global resolvability of an initial value problem with double nonlinearity and qualitative properties of the solution of a task on the basis of the self-similar analysis are investigated. These qualitative properties of the considered task it carried out because of a research of qualitative properties of the self-similar equation.

Let's consider in the domain $\mathrm{Q}=\{(\mathrm{t},\mathrm{x}):\ 0<\mathrm{t},\mathrm{x}\in\mathrm{R}\}$ quasilinear equation of reaction diffusion with double nonlinearity

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(Du^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right)+ku\left(1-u^{\beta}\right),\ \ u\big|_{t=0}=u_0(x)\ge 0,\ x\in R \tag{11}$$

which describes reaction diffusion process which diffusion coefficients are equal $D\,u^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}$, where m, p, β - positive real numbers, $u=u(t,x)\ge 0$ - required solution.

Let's note that the main feature in studying of nonlinear properties is receiving various types of estimates of solutions, and then on their basis numerical model operation of a task. In it the larger role played by the self-similar and approximate and self-similar approaches that are widely presented in particular in A.A. Samarsky, S.P. Kurdyumov, V.A. Galaktionov, A.P. Mikhaylov's works [16].

For creation of the self-similar equation, the algorithm of nonlinear splitting is used [17].

Let's note that replacement in (11) (11) $u(t,x)=e^{kt}v(\tau(t),x)$ will lead (11) to a look:

$$\frac{\partial v}{\partial\tau}=\frac{\partial}{\partial x}\left(Dv^{m-1}\left|\frac{\partial v}{\partial x}\right|^{p-2}\frac{\partial v}{\partial x}\right)-ke^{[(\beta-(m+p-3)k]t}v^{\beta+1}, \tag{12}$$

$$v\big|_{t=0}=v_0(x)=u_0(x).$$

where $\tau(t)=e^{[(m+p-3)k]t}/(m+p-3)k,\ m+p-3\neq 0$.

Taking into account (3) equation (2) will correspond in the following look:

$$\frac{\partial v}{\partial \tau}=\frac{\partial}{\partial x}\left(Dv^{m-1}\left|\frac{\partial v}{\partial x}\right|^{p-2}\frac{\partial v}{\partial x}\right)-k_1\tau^b v^{\beta+1}, \tag{13}$$

where $k_1=k\left((m+p-3)k\right)^b$, $b=\frac{(\beta-(m+p)-3)}{m+p-3}$.

For the purpose of receiving the self-similar equation we will apply a splitting algorithm to the equation (13) according to which we will find at first the solution of an ordinary differential equation $\frac{d\bar{v}}{d\tau}=-k_1\tau^b\bar{v}^{\beta+1}$,

Then we have $\bar{v}(\tau)=c(\tau+T_0)^{-\gamma}$, $T_0>0$, where $c=\left[\frac{\beta k_1}{b+1}\right]^{-\frac{1}{\beta}}$, $\gamma=\frac{b+1}{\beta}$.

Then the solution of the equation (13) looked for in a look $v(t,x)=\bar{v}(t)w(\tau,x)$, a function $\tau=\tau(t)$ is chosen so

$$\tau(t)=\int_0^t \bar{v}^{(m+p-3)}(t)dt=\begin{cases} t^{1-[\gamma(m+p-3)]}/(1-\gamma(m+p-3)), & if\ \ 1-\gamma(m+p-3)\neq 0,\\ \ln t, & if\ \ 1-\gamma(m+p-3)=0,\\ t, & if\ \ m+p=3. \end{cases}$$

Then for $w(\tau,x)$ let's receive the equation

$$\frac{\partial w}{\partial \tau}=\frac{\partial}{\partial x}\left(Dw^{m-1}\left|\frac{\partial w}{\partial x}\right|^{p-2}\frac{\partial w}{\partial x}\right)+\psi(\mathrm{w}-w^{\beta+1}), \tag{14}$$

where

$$\psi=\begin{cases} \dfrac{1}{(1-[\gamma(m+p-3)])\tau}, & if\ \ 1-[\gamma(m+p-3)>0,\\ \gamma c^{-(\gamma(m+p-3))}, & if\ \ 1-[\gamma(m+p-3)=0. \end{cases} \tag{15}$$

That shows an invariance of the transformation given above. Let us consider the self-similar solution

$$w(\tau,x)=f(\xi),\quad \xi=|x|/\tau^{1/p}\ . \tag{16}$$

for equation (14).

Then substituting (16) in (14) in the case $1-[\gamma(m+p-3)>0$, regarding to $f(\xi)$ it is easy to receive the self-similar equation:

$$L(f)=\frac{d}{d\xi}(f^{m-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi})+\frac{\xi}{p}\frac{df}{d\xi}+\mu(f-f^{\beta+1})=0,\ \mu=\frac{1}{1-[\gamma(m+p-3)]}. \tag{17}$$

Let's be engaged in creation of the upper solution for the equation (11). If $\beta = [3-(p+m)]/(p-1)$, then equation (17) has the exact solution of a look $\overline{f}_{\pm}(\xi) = A(a \pm \xi^{\gamma})_{+}^{n}$

where $n = (p-1)/(p+m-3)$, $\gamma = p/(p-1)$, $(b)_{+} = \max(0,b)$ [19].

Properties of solutions of equation (17) in the case *p=2, m=1*investigated in detail in [3].

Theorem 1

Let $u(0,x) \le z_{\pm}(0,x),\ x \in R.$ Then for the solution of problem (11) in the domain Q the estimate $u(t,x) \le z_{\pm}(t,x) = (T+t)^{-\gamma}\,\overline{f}_{\pm}(\xi)$, $\xi = |x| / \tau^{1/p}$ Here $\overline{f}_{\pm}(\xi)$ function defined above.

3. RESEARCH OF PROPERTIES OF SOLUTIONS OF CROSS-DIFFUSION MODEL OF REACTION DIFFUSION WITH DOUBLE NONLINEARITY

Consider in the domain Q={(t,x): 0<t<∞, x∈ R ^{N} }parabolic system of cross-diffusion

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \nabla\left(|x|^n \left|\nabla u_1^k\right|^{p-2} \nabla u_2^{m_1}\right) + k_1\left(u_1 - u_1^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t} = \nabla\left(|x|^n \left|\nabla u_2^k\right|^{p-2} \nabla u_1^{m_2}\right) + k_1\left(u_2 - u_2^{\beta_2}\right), \end{cases} \tag{18}$$

$$u_1\big|_{t=0} = u_{10}(x),\ u_2\big|_{t=0} = u_{20}(x),$$

which describes process of reaction-diffusion in the nonlinear bipropellant environment which diffusion coefficients are equal $|x|^n \left|\nabla u_1^k\right|^{p-2}, |x|^n \left|\nabla u_2^k\right|^{p-2}$; numeric parameters $m_1, m_2, n, p, \beta_1, \beta_2$ - positive real numbers, $\beta_1, \beta_2 \ge 0$, $u_1 = u_1(t,x) \ge 0$, $u_2 = u_2(t,x) \ge 0$ - density.

In this work it is investigated properties of solutions of an initial value problem for the system of biological population with double nonlinearity. The main research technique is self-similar approach. We will construct a self-similar set of equations by method of nonlinear splitting [17].

Substitution in (18) $u_1(t,x) = e^{-k_1 t} v_1(\tau(t),x),\quad u_2(t,x) = e^{-k_2 t} v_2(\tau(t),x)$ will lead (18) to the form:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \nabla\left(|x|^n \left|\nabla v_1^k\right|^{p-2} \nabla v_2^{m_1}\right) - a_1 \tau^{b_1} v_1^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \nabla\left(|x|^n \left|\nabla v_2^k\right|^{p-2} \nabla v_1^{m_2}\right) - a_2 \tau^{b_2} v_2^{\beta_2}, \end{cases} \tag{19}$$

$$v_1\big|_{t=0} = v_{10}(x),\ v_2\big|_{t=0} = v_{20}(x).$$

Here

$$\tau(t) = \frac{e^{[m_1 k_2 + (p-2)kk_1 - k_1]t}}{m_1 k_2 + (p-2)kk_1 - k_1} = \frac{e^{[m_2 k_1 + (p-2)kk_2 - k_2]t}}{m_2 k_1 + (p-2)kk_2 - k_2},$$

$$b_1=\frac{k_1\beta_1-(p-2)kk_1-m_1k_2}{m_1k_2+(p-2)kk_1-k_1},\ b_2=\frac{k_2\beta_2-(p-2)kk_1-m_2k_1}{m_2k_1+(p-2)kk_1-k_2}.$$

Then the solution of system (19) is sought in the form

$$v_1(t,x)=\bar{v}_1(\tau)w_1(\tau(t),\varphi(|x|)),\ v_2(t,x)=\bar{v}_2(\tau)w_2(\tau(t),\varphi(|x|)) \quad ,$$

$$\bar{v}_1(\tau)=(T_0+\tau)^{-\gamma_1},\ \bar{v}_2(\tau)=(T_0+\tau)^{-\gamma_2},\ T_0>0, \tag{20}$$

Where at $b_1=0,\ b_2=0:\ \gamma_1=\dfrac{1}{\beta_1-1},\ \gamma_2=\dfrac{1}{\beta_2-1}$, at $b_1\neq 0,\ b_2\neq 0:\ \gamma_1=\dfrac{b_1+1}{\beta_1-1},\ \gamma_2=\dfrac{b_2+1}{\beta_2-1}$.

Then for $w_i(\tau,\varphi(|x|)),\ i=1,2$ get the system of equations:

$$\begin{cases}\dfrac{\partial w_1}{\partial\tau}=\varphi^{1-s}\dfrac{\partial}{\partial\varphi}\left(\varphi^{s-1}\left|\dfrac{\partial w_1^k}{\partial\varphi}\right|^{p-2}\dfrac{\partial w_2^{m_1}}{\partial\varphi}\right)+\psi_1(w_1-w_1^{\beta_1}),\\[2ex] \dfrac{\partial w_2}{\partial\tau}=\varphi^{1-s}\dfrac{\partial}{\partial\varphi}\left(\varphi^{s-1}\left|\dfrac{\partial w_2^k}{\partial\varphi}\right|^{p-2}\dfrac{\partial w_1^{m_2}}{\partial\varphi}\right)+\psi_2(w_2-w_2^{\beta_2}),\end{cases} \tag{21}$$

where $\psi_1=\dfrac{1}{(1-\gamma_1[(p-2)k-1]-\gamma_2m_1)\tau_1}$ and $\psi_2=\dfrac{1}{(1-\gamma_2[(p-2)k-1]-\gamma_1m_2)\tau_2}$.

Here at $p>n:\ \varphi(|x|)=|x|^{p_1}/p_1,\ p_1=(p-n)/p,\ s=pN/(p-n)$,

and at $p=n:\ \varphi(|x|)=\ln(|x|)$,.

Self-similar solution of system (21) looks like

$$w_i(\tau(t),\varphi)=f_i(\xi),\ \xi=\varphi(|x|)/\tau^{1/p}. \tag{22}$$

Then substituting (22) in (21) relatively $f_i(\xi)$ get a system of self-similar equations

$$\begin{cases}\xi^{1-s}\dfrac{d}{d\xi}\left(\xi^{s-1}\left|\dfrac{df_1^k}{d\xi}\right|^{p-2}\dfrac{df_2^{m_1}}{d\xi}\right)+\dfrac{\xi}{2}\dfrac{df_1}{d\xi}+\mu_1(f_1-f_1^{\beta_1})=0,\\[2ex] \xi^{1-s}\dfrac{d}{d\xi}\left(\xi^{s-1}\left|\dfrac{df_2}{d\xi}\right|^{p-2}\dfrac{df_1^{m_2}}{d\xi}\right)+\dfrac{\xi}{2}\dfrac{df_2}{d\xi}+\mu_2(f_2-f_2^{\beta_2})=0.\end{cases} \tag{23}$$

where $\mu_1=\dfrac{1}{(1-\gamma_1[(p-2)k-1]-\gamma_2m_1)}$ и $\mu_2=\dfrac{1}{(1-\gamma_2[(p-2)k-1]-\gamma_1m_2)}$.

The system (23) has approximate solution of a look

$$\bar{f}_1=A(a-\xi)_+^{n_1},\ \bar{f}_2=B(a-\xi)_+^{n_2},$$

where

$$n_1 = \frac{(p-1)[\mathrm{k}(p-2)-m_1]}{[k(p-2)]^2 - m_1 m_2}, n_2 = \frac{(p-1)[\mathrm{k}(p-2)-m_2]}{[k(p-2)]^2 - m_1 m_2}.$$

Theorem

Let $u_i(0,x) \le z_i(0,x),\ x \in R^N,\ \ \mathrm{i}=1,2$ Then for the solution of a task (18) in area Q $u_1(t,x) \le z_1(t,x) = (T+t)^{-\gamma_1}\,\overline{f}_1(\xi),\ \ u_2(t,x) \le z_2(t,x) = (T+t)^{-\gamma_2}\,\overline{f}_2(\xi),\ \xi = \varphi(|x|)/\tau^{1/p}$ assessment takes place.

Here $\overline{f}_i(\xi)$ the functions defined above.

In case $n_1 > 0, n_2 > 0,\ q > 0$ applying a method [17] to the solution of the equation (23) we will receive the following functions

$$\overline{\theta}_1(\xi) = (a-\xi)_+^{\ n_1},\ \overline{\theta}_2(\xi) = (a-\xi\ )_+^{\ n_2},$$

Theorem

Limited solution of a system (23) at $\xi \to a\_$ has asymptotics $f_i(\xi) \sim \vartheta_i(\xi)$.

In case $n_1 > 0, n_2 > 0,\ q < 0$ for (23) we have

$$\chi_1(\xi) = (a+\xi\ )^{n_1},\ \chi_2(\xi) = (a+\xi\ )^{n_2},$$

where $a > 0$, $\mathrm{q} = [k(p-2)-1]^2 - m_1 m_2$.

Theorem

At $\xi \to +\infty$ task solution (23) disappearing on infinity the solution has an asymptotics $f_i(\xi) \sim \chi_i(\xi),\quad i = 1,2$.

4. CONCLUSIONS

The properties stated above because of the theorem comparison of the solution established, an asymptotics of self-similar solutions, including for a case of fast diffusion received. Based on the found solutions, numerical calculations carried out. The research of qualitative properties of a system (18) allowed, to execute a numerical experiment depending on values, the logging-in numerical parameters. For this purpose as an initial approximation an asymptotics of solutions used.

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APPENDIX

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \nabla\left(|x|^n \left|\nabla u_1^k\right|^{p-2} \nabla u_2^{m_1}\right) + k_1\left(u_1 - u_1^{\beta_1}\right) \\ \dfrac{\partial u_2}{\partial t} = \nabla\left(|x|^n \left|\nabla u_2^k\right|^{p-2} \nabla u_1^{m_2}\right) + k_1\left(u_2 - u_2^{\beta_2}\right) \end{cases}$$

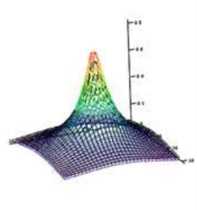
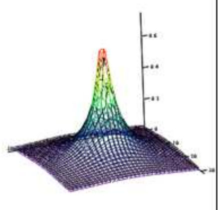
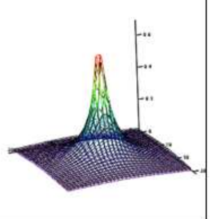
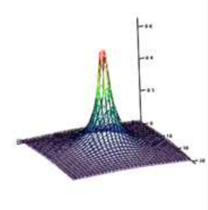
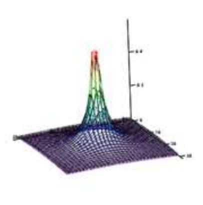
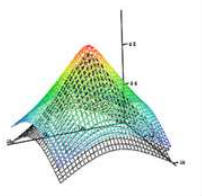
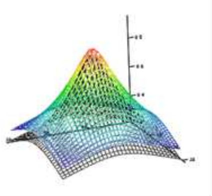
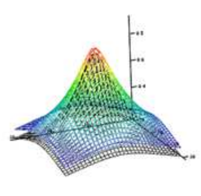
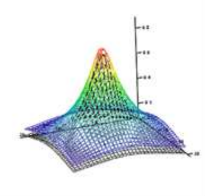
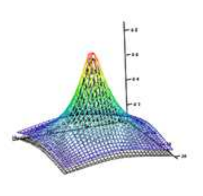
- Fast diffusion. As an initial approximation, it is necessary to take:

$$u_1(x,t) = (T+\tau(t))^{-\gamma_1}(a+\xi^{\gamma})^{n_1}, u_2(x,t) = (T+\tau(t))^{-\gamma_2}(a+\xi^{\gamma})^{n_2}, \gamma_1 = \frac{1}{\beta_1 - 1}, \gamma_2 = \frac{1}{\beta_2 - 1},$$

$\gamma = \dfrac{p}{p-1}, \xi = \varphi(|x|)/\tau^{1/p}$, $p > n : \varphi(|x|) = |x|^{p_1}/p_1, p_1 = (p-n)/p, n_1 = \dfrac{(p-1)[k(p-2)-m_1]}{q},$

$n_2 = \dfrac{(p-1)[k(p-2)-m_2]}{q},\ q = [k(p-2)]^2 - m_1 m_2$, $n_1 > 0, n_2 > 0, q < 0$

Table 1

Values of Parameters	$t_{\max} = 1, x_{1\max} = 0.972, x_{2\max} = 0.972$	$t_{\max} = 2, x_{1\max} = 1.046, x_{2\max} = 1.046$	$t_{\max} = 3, x_{1\max} = 1,096, x_{2\max} = 1.096$	$t_{\max} = 4, x_{1\max} = 1.132, x_{2\max} = 1.132$	$t_{\max} = 5, x_{1\max} = 1.162, x_{2\max} = 1.162$
$m_1 = 8, m_2 = 7, p = 3$ $eps = 10^{-3}$ $n_1 = 0.255 > 0$ $n_2 = 0.218 > 0$ $q = -55 < 0$ $\beta_1 = 2\ \ \beta_2 = 3$ $k = 2$ n=0.5					
$m_1 = 5.9, m_2 = 7\ p = 3$ $eps = 10^{-3}$ $n_1 = 0.312 > 0$ $n_2 = 0.365 > 0$ $q = -41.05 < 0$ $\beta_1 = 2,\ \beta_2 = 3$ $k = 0.5$ n=0.3					

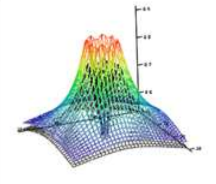
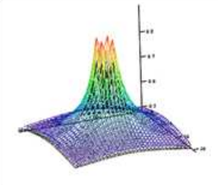
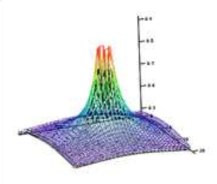
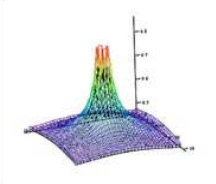
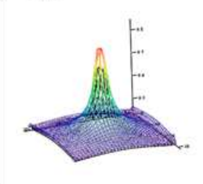
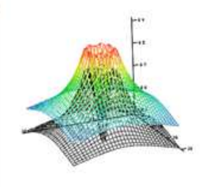
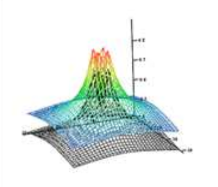
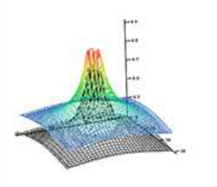
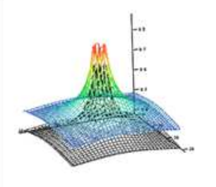
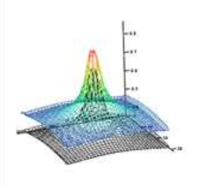
- Fast diffusion. As an initial approximation, it is necessary to take

$$u_1(x,t) = (T+\tau(t))^{-\gamma_1}(a+\xi^{\gamma})^{n_1}, u_2(x,t) = (T+\tau(t))^{-\gamma_2}(a+\xi^{\gamma})^{n_2}, \gamma_1 = \frac{1}{\beta_1 - 1}, \gamma_2 = \frac{1}{\beta_2 - 1},$$

$\gamma = \dfrac{p}{p-1}, \xi = \varphi(|x|)/\tau^{1/p}$, $p = n :\quad \varphi(|x|) = \ln(|x|)$, $n_1 = \dfrac{(p-1)[k(p-2)-m_1]}{q}, n_2 = \dfrac{(p-1)[k(p-2)-m_2]}{q},$

$q = [k(p-2)]^2 - m_1 m_2$, $n_1 > 0, n_2 > 0, q < 0$

Table 2

Values of Parameters	$t_{max}=1,\ x_{1max}=2.752,\ x_{2max}=2.752$	$t_{max}=2,\ x_{1max}=3.152,\ x_{2max}=3.152$	$t_{max}=3,\ x_{1max}=3.476,\ x_{2max}=3.476$	$t_{max}=4,\ x_{1max}=3.742,\ x_{2max}=3.742$	$t_{max}=5,\ x_{1max}=3.974,\ x_{2max}=3.974$
$m_1=8, m_2=7, p=3$ $eps=10^{-3}$ $n_1=0.255>0$ $n_2=0.218>0$ $q=-55<0$ $\beta_1=2\ \beta_2=3\ k=2$ n=3					
$m_1=5.9, m_2=7\ p=3$ $eps=10^{-3}$ $n_1=0.312>0$ $n_2=0.365>0$ $q=-41.05<0$ $\beta_1=2,\ \beta_2=3$ $k=0.5$ n=3					

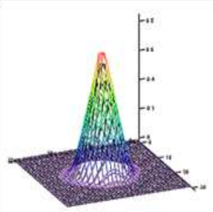
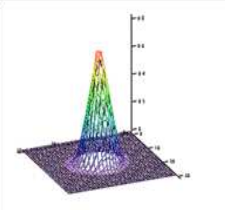
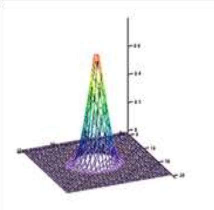
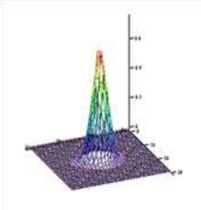
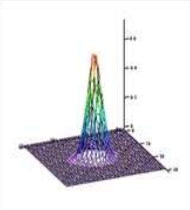
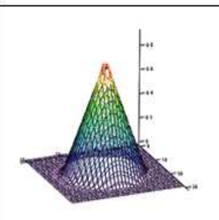
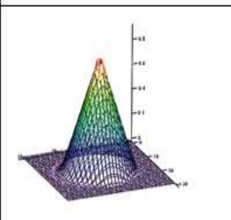
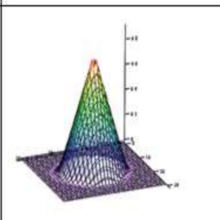
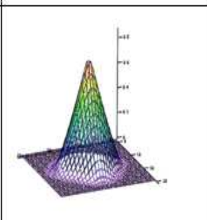
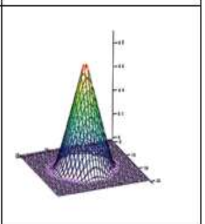
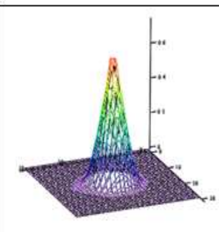
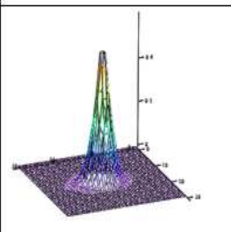
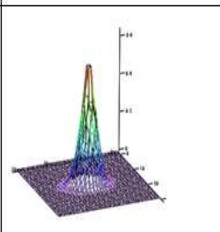
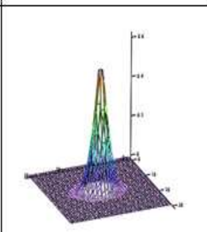
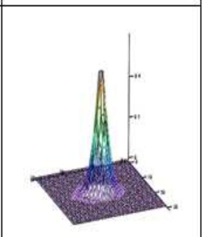
- Slow diffusion. As an initial approximation, it is necessary to take:

$$u_1(x,t)=(T+\tau(t))^{-\gamma_1}(a-\xi^{\gamma})_+^{\ n_1}, \quad u_2(x,t)=(T+\tau(t))^{-\gamma_2}(a-\xi^{\gamma})_+^{\ n_2}, \quad \gamma_1=\frac{1}{\beta_1-1}, \quad \gamma_2=\frac{1}{\beta_2-1},$$

$$\gamma=\frac{p}{p-1}, \xi=\varphi(|x|)/\tau^{1/p},\ p>n:\varphi(|x|)=|x|^{p_1}/p_1,\ p_1=(p-n)/p,$$

$$n_1=\frac{(p-1)[k(p-2)-m_1]}{q}, n_2=\frac{(p-1)[k(p-2)-m_2]}{q},\ q=[k(p-2)]^2-m_1m_2,\ n_1>0, n_2>0, q>0.$$

Table 3

Values of Parameters	$t_{max}=1,\ x_{1max}=0.972,\ x_{2max}=0.972$	$t_{max}=2,\ x_{1max}=1.046,\ x_{2max}=1.046$	$t_{max}=3,\ x_{1max}=1{,}096,\ x_{2max}=1.096$	$t_{max}=4,\ x_{1max}=1.132,\ x_{2max}=1.132$	$t_{max}=5,\ x_{1max}=1.162,\ x_{2max}=1.162$
$m_1=0.5, m_2=0.7, p=5$ $eps=10^{-3}$ $n_1=0.73>0$ $n_2=0.698>0$ $q=24.65>0$ $\beta_1=2,\ \beta_2=3$ $k=2$, n=0.5					
$m_1=2, m_2=4,\ p=10$ $eps=10^{-3}$ $n_1=1.242>0$ $n_2=0.65$ $q=30.44>0$ $\beta_1=5,\ \beta_2=7$ $k=0.9$, n=0.5					
$m_1=0.2, m_2=1.5,\ p=7$ $eps=10^{-3}$ $n_1=0.654>0$ $n_2=0.558>0$ $q=80.7>0$ $\beta_1=2,\ \beta_2=3$ $k=2$ n=2					

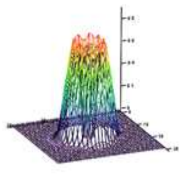
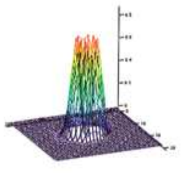
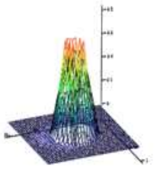
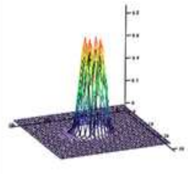
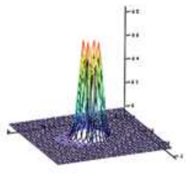
- Slow diffusion. As an initial approximation, it is necessary to take

$$u_1(x,t) = (T+\tau(t))^{-\gamma_1}(a-\xi^{\gamma})_+^{n_1}, \quad u_2(x,t) = (T+\tau(t))^{-\gamma_2}(a-\xi^{\gamma})_+^{n_2}, \quad \gamma_1 = \frac{1}{\beta_1 - 1}, \quad \gamma_2 = \frac{1}{\beta_2 - 1},$$

$$\gamma = \frac{p}{p-1}, \xi = \varphi(|x|)/\tau^{1/p}, \; p = n: \; \varphi(|x|) = \ln(|x|),$$

$$n_1 = \frac{(p-1)[k(p-2)-m_1]}{q}, n_2 = \frac{(p-1)[k(p-2)-m_2]}{q}, \; q = [k(p-2)]^2 - m_1 m_2, \; n_1 > 0, n_2 > 0, q > 0.$$

Table 4

Values of Parameters	$t_{\max}=1$, $x_{1\max}=2.752$, $x_{2\max}=2.752$	$t_{\max}=2$, $x_{1\max}=3.152$, $x_{2\max}=3.152$	$t_{\max}=3$, $x_{1\max}=3.476$, $x_{2\max}=3.476$	$t_{\max}=4$, $x_{1\max}=3.742$, $x_{2\max}=3.742$	$t_{\max}=5$, $x_{1\max}=3.974$, $x_{2\max}=3.974$
$m_1=0.5$, $m_2=0.7$, $p=5$ $eps=10^{-3}$ $n_1=0.73>0$ $n_2=0.698>0$ $q=24.65>0$ $\beta_1=2$, $\beta_2=3$ $k=2$, n=5					
$m_1=2$, $m_2=4$, $p=10$ $eps=10^{-3}$ $n_1=1.242>0$ $n_2=0.65$ $q=30.44>0$ $\beta_1=5$, $\beta_2=7$ $k=0.9$, n=10